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EXTENDED ABSTRACT

Use of Stochastic Differential Equations in Investigating the Uncertainties Related to the Operation of the Activated Sludge Wastewater Treatment Plant

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1. Introduction

In the present paper, the uncertainty analysis for the activated sludge part in the wastewater treatment plant (WWTP) was done using stochastic differential equation (SDE) equations. Euler-Maruyama method was selected for the numerical solution of the Itˆo integral and the results were compared and analyzed with observational data.

2. Methodology

2.1. Case Study area and data

The wastewater treatment plant of Tabriz city was selected as a study case, its location is shown in Fig. 1. The data of the 15-day period during the year (2019-2020) was considered for analysis.

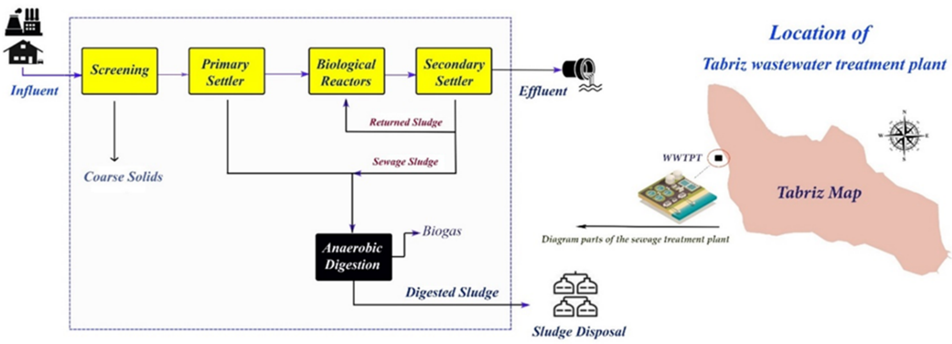

Fig. 1. Location and performance diagram of Tabriz wastewater treatment plant

In this research, uncertainty was investigated on pollution indicators such as BOD and *Q* for the biological section of WWTP. The statistical data for the variables of the biological sector and the behavior of the changes

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in the input and output flow to the WWTP during a one-year period (2019-2020) in 15 day's were recorded and analyzed.

2.2. Activated sludge process and SDE equations

Biological processes are necessary for the removal of organic matter, colloidal solution and biological nitrification and biological phosphorus removal. The main purpose of this unit is to separate a large volume of mixed liquor suspended solids (MLSS) from the aeration pool and produce a clear and stable flow with low concentration of total suspended solids (TSS) and biochemical oxygen demand (BOD). The behavior of the activated sludge process can be investigated with differential equations. but the investigation of uncertainty for the pollution parameters of the biological treatment section with SDE would certainly complement ODE. Therefore, in this article, SDE has been used to investigated the uncertainties related to the activated sludge part and the biological parameters of the output or inside the system. The performance of calculations is shown schematically in Fig. 2.

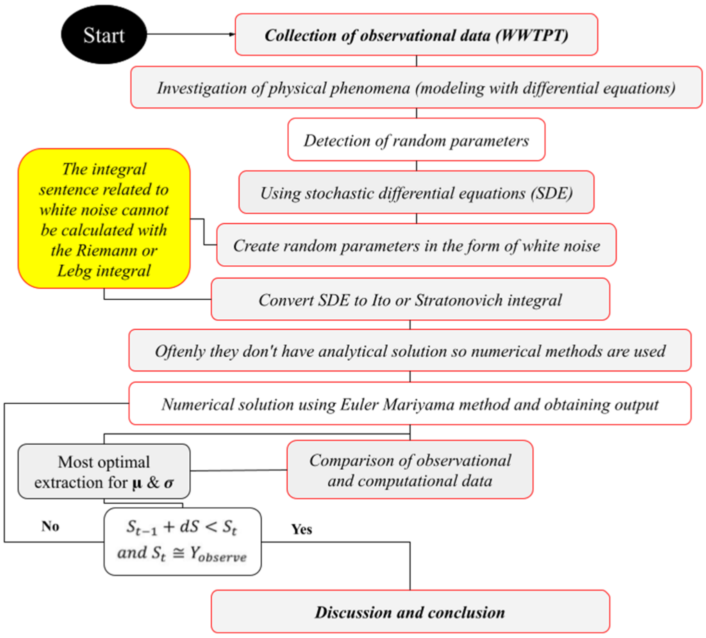


Fig. 2. Diagram of solving SDE equations

3. Results and discussion

3.1. SDE Extraction for Activated Sludge

The probability space for a random phenomenon is a Wiener process with a continuous path in a certain time step with $W(t_i)$ independent variables. Due to the presence of sentences in terms of wiener processes in the corresponding stochastic integral, their solution is not derivable in terms of t, and usually these types of equations are expressed in the form of stochastic integral $I\hat{t}o$. The generalization of the law of conservation of mass and continuity for the activated sludge section follows from equation (1).

$$\forall \frac{d\mathcal{X}}{dt} = Qy(S_0 - S_e) + Q_w\mathcal{X}_u + k_d\forall\mathcal{X} \tag{1}$$

So that $\mathcal{X}$ biomass concentration (mg/lit), $\forall$ reactor volume (m3), S_e output substrate concentration at time t (mg/lit), S_0 input substrate concentration at time t_0, Q_w wastewater flow back to the biological reactor with concentration $\mathcal{X}_u$, The stochastic form of activated sludge equations can be seen in equations (2) and (3). k_d is the weight of the biological mass that is lost per unit of time (day^{-1}). Using the Euler-Maruyama method and using geometric Brownian motion with log-normal distribution, the stochastic equations for flow rate and sludge concentration changes are obtained as relations (2) and (3) which are chosen by trial and error.

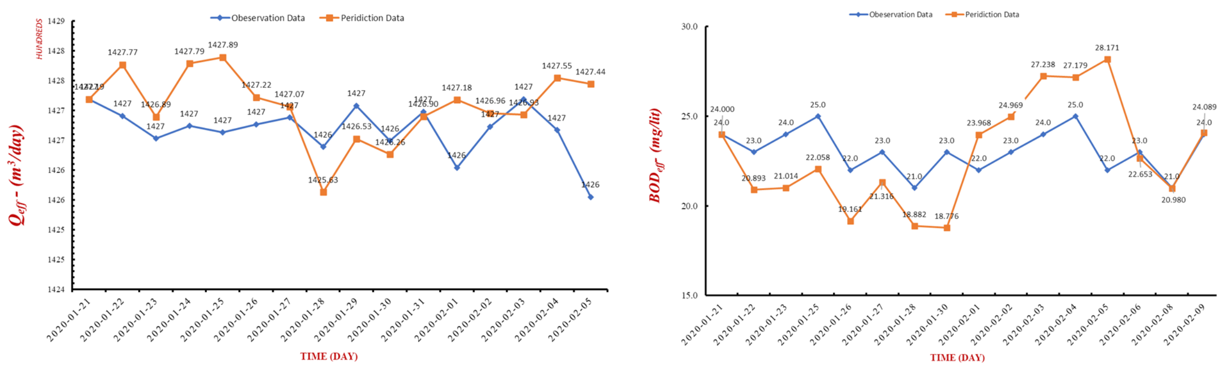
$$dQ = a_1(\bar{Q} - Q_t)dt + c_1\sigma_Q\sqrt{Q_t}dW_1 \quad (2)$$

$$dS_0 = a_2(\bar{S} - S_e)dt + c_2\sigma_{S_0}\sqrt{S_e}dW_2 \quad (3)$$

So that σ standard deviation is preferable to variance because it is the same dimension as observational data. dW_i random independent variables in the form $\sqrt{\delta t}N(0,1)$ for Brownian motion with distribution function between intervals (0,1). a_i & c_i constant coefficients.

3.2. SDE modeling results for Q

After deriving the SDE equations for the activated sludge part of WWTP, the results of constant coefficients a_i and c_i for *Q_{eff}* were found to be 0.0035 and 0.0028, respectively, with δt equal to 0.001. The comparison chart of observed values with calculations in Fig. (3-a) it has been shown. The convergence of the numerical results of the SDE equations for BOD_{eff} was calculated as 0.005 and 0.7 with δt equal to 0.001, which is shown in Fig. (3-b).



(a) (b)

Fig. 3. a) Modeling Qeff with SDE for observed-computed data, **b)** modeling BODeff with SDE for observed-computed data

4. Conclusions

Nowadays, the investigation of uncertainty in physical and engineering phenomena and the use of SDE have been more attention of researchers. Therefore, the analysis of the activated sludge process was carried out using these equations and monitoring observing changes in the pollution parameters, Qeff and BODeff, as a biological process after extracting the relevant equations for a period of 15 days from the data (2019-2020). The period of 15 days was chosen to prevent the entry of wrong information due to cumulative errors. The results show the possibility of monitoring the fluctuations of the biological part of WWTP with SDE. One of the advantages of this method is reducing economic costs and observing changes in short periods of time.