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EXTENDED ABSTRACT

Analysis of Longitudinal Wave Propagation in Functionally Graded Materials (FGMs) Using Wave Element Method

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1. Abstract

This paper investigates the complex phenomenon of longitudinal wave propagation in functionally graded materials (FGMs). Unlike traditional homogeneous materials with constant properties, FGMs exhibit a continuous variation in properties across their structure. This inherent inhomogeneity presents unique challenges for analyzing wave propagation behavior. The study employs the wave element method, a powerful numerical technique commonly used for solving dynamic problems in complex structures. This method allows for the effective analysis of wave propagation in FGMs by incorporating the gradual variations in material properties along the length of the rod. The obtained results reveal significant differences in wave propagation between FGMs and homogeneous materials. In FGMs, wave characteristics like velocity, wavelength, and amplitude continuously vary along the material. This distinct behavior can be directly attributed to the inhomogeneity of material properties within the FGM. This finding underscores the importance of considering material property variations when analyzing wave propagation in FGMs compared to their homogeneous counterparts.

2. Introduction

Among the fundamental issues involved in wave propagation, homogeneous plane waves are considered to be an important component. In a homogeneous, linear, and non-dispersive solid, the Helmholtz equation describes these waves as amplitudes (A) and vectors (k), which are real-valued representations of a wave propagating in a homogeneous, linear, and non-dispersive solid. However, for materials exhibiting (linear) dispersive behavior, the simplest wave form deviates from this ideal state. While the wave retains a similar structure, the wave amplitude (A) and wave vector (k) acquire complex values. This shift from real to complex values necessitates a distinction between them. In such cases, the term "inhomogeneous wave" is commonly used. In principle, the term "inhomogeneous wave" can have two meanings. In the context of material properties, it denotes spatial variations; in the context of wave propagation through dissipative media, it represents a complex k-vector and associated amplitude change. Recognizing this duality will lead to a deeper understanding of wave behavior in various material systems. Viscoelastic materials, characterized by inherent energy dissipation mechanisms, are the primary setting for the formation of inhomogeneous waves. Extensive research has been conducted on these waves, demonstrating their effectiveness in explaining various wave modes and their dispersive nature (Gavassino et al., 2024). In dispersive media, the propagation velocity, attenuation constant, and associated angles are all frequency-dependent. In addition to viscoelastic materials, another class of environments that support inhomogeneous wave propagation is functionally graded materials (FGMs). Unlike homogeneous specimens with uniform properties throughout, FGM materials possess spatially varying elastic and inertial properties. This variation can occur continuously and be tailored to achieve specific performance objectives. In nature, most materials exhibit some degree of inhomogeneity at the microscopic

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level. However, for many engineering applications, they are often treated as homogeneous materials due to simplicity. Traditionally, metals and other industrial materials have largely been considered homogeneous, with their inherent defects neglected. However, recent advancements have enabled the design and fabrication of FGM materials with precisely controlled property variations in space.

3. Problem Formulation

This section introduces the governing equations for wave propagation in a one-dimensional (1D) inhomogeneous waveguide. Considering a rod with spatially varying properties of material along its length and thickness (as shown in Figure 1). This simplified model depicts a 1D waveguide that includes three distinct sections made up of three different materials: steel, functionally graded material (FGM), and ceramic. It is assumed that the properties of steel and ceramic sections are independent of the spatial coordinates of the sections in question. The FGM section on the other hand is distinguished from the homogeneous section due to the gradual change in properties over the length of the section (x-direction) for both the elastic modulus (E) and density (ρ).

3. FGM Modeling

To model the inhomogeneous nature of the FGM section, different material property variation laws can be employed. Two distinct approaches can be considered: polynomial variation and exponential variation. For the polynomial variation, the functions representing the elastic modulus and density are expressed as polynomials of the rod's length. This approach allows for a gradual and controlled change in material properties within the FGM section. The alternative approach employs exponential variation as a means of achieving a more rapid transition within the FGM as opposed to the linear variation. The method selected in this paper is the polynomial variation method.

3. Wave Element Method

This section presents a numerical method called the wave element method. This method is based on the discretization of a rod of length L into elements of arbitrary lengths by means of a discretization process. According to this method, the propagation of strong discontinuities in velocities, stresses, and strains in terms of finite wave velocity is used to describe the propagation of discontinuities (Mirzajani et al., 2018; 2021).

4. Results and Discussion

The simulation results reveal significant differences in wave propagation between FGMs and homogeneous materials. In FGMs, wave velocity, wavelength, and amplitude continuously vary along the material's length due to the inherent inhomogeneity of the material properties.

Based on the findings, the absence of reflection and the preservation of the input wave's shape suggest that the impulse wave propagates uniformly along the rod. This points out the rod's ability to function as a single, cohesive unit during wave propagation. This behavior is attributed to the linear material properties of the rod, which allow for the consistent transmission of mechanical energy without significant attenuation or distortion. Therefore, we can conclude that this type of structure behaves as a whole in opposition to the applied impulse wave.

5. Conclusions

This study shows that the propagation of longitudinal waves in FGMs is a complex phenomenon influenced by the inhomogeneity of material properties and the type of property variation law. The wave element method presented in this paper is an effective tool for analyzing wave propagation in FGM environments. This method holds potential for the design of industrial components, such as aircraft wings or turbine blades, where maintaining structural integrity during wave propagation is critical.

6. References

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