

## EXTENDED ABSTRACT

# Uncertainty Analysis of Stage-Discharge Rating Curves In Rivers

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## 1. Introduction

The primary purpose of this paper is to analyze the uncertainty by assessing gauging deviations of the stage-discharge rating curve in rivers. In this research, the basis of the estimation of the rating curve is the concept of isovel contours in the Single Point Measurement method (SPM). Observed data in the Nazli-chai River in Iran, Main River in England and Colorado River in Argentina are used to investigate the global uncertainty in the rating curve estimation. The results can be used to improve flood control and water resource management.

## 2. Methodology

### 2.1. The proposed stage-discharge relationship

Maghrebi et al. (2017) assumed that the discharge at any stage of a channel could be stated as a function of the following parameters:

$$Q = f(A, P, T, P_t, U_{SPM}, n, S_0) \quad (1)$$

Where  $Q$  is the discharge,  $A$  is the cross-section area,  $P$  is the wetted perimeter of the flow section,  $P_t$  is the sum of  $P$  and the width of the water surface ( $P_t = P + T$ ),  $U_{SPM}$  is the cross-sectional mean flow velocity in the stream-wise direction,  $n$  is the Manning roughness and  $S_0$  is a longitudinal bed slope. According to Eq. 1, a general form of the stage-discharge relationship is as follows:

$$\frac{Q_e}{Q_r} = \left(\frac{A_e}{A_r}\right)^{a_1} \left(\frac{P_e}{P_r}\right)^{a_2} \left(\frac{(P_t)_e}{(P_t)_r}\right)^{a_3} \left(\frac{(U_{SPM})_e}{(U_{SPM})_r}\right)^{a_4} \left(\frac{n_e}{n_r}\right)^{a_5} \left(\frac{(S_0)_e}{(S_0)_r}\right)^{a_6} \quad (2)$$

Where the subscripts  $r$  and  $e$  refer to the referenced and estimated values, respectively. The value of  $a_6$  is set to zero because the effect of the bed slope of the channel, which stays fixed at all water levels, can be ignored in the computational processing. Maghrebi et al. (2017) have presented the most reliable relationship, which is associated with the least values of NRMSE. They have suggested their last relationship as follows:

$$Q_e = Q_r \left(\frac{A_e}{A_r}\right)^{0.972} \left(\frac{P_e}{P_r}\right)^{-1.27} \left(\frac{(P_t)_e}{(P_t)_r}\right)^{0.83} \left(\frac{(U_{SPM})_e}{(U_{SPM})_r}\right) \left(\frac{n_e}{n_r}\right)^{-1} \quad (3)$$

In order to estimate the discharge by the Eq. 3, all of the effective parameters are needed to be calculated at all water levels in the range of the required rating curve.

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## 2.2. The global uncertainty analysis method

Di Baldassarre and Montanari (2009) observed that the main sources of error affecting the rating curve are the error in discharge measurement ( $\varepsilon_1$ ) and the error caused by the uncertainty of the rating curves ( $\varepsilon_2$ ). The authors assumed that global uncertainty could be obtained as follows:

$$\varepsilon(Q(x,t)) = \pm\varepsilon_1(Q(x,t)) + \varepsilon_2(Q(x,t)) \quad (4)$$

In this study,  $\varepsilon_1$  is considered as a normal random variable that mean value is zero, and the standard deviation is  $0.027Q_r$ . Therefore, it is possible to obtain the confidence interval of the observed data ( $Q_r$ ) values from Eq. 5:

$$Q_r \pm \left\{ \alpha \times 0.027Q_r + |\varepsilon_2(Q_r)| \right\} = Q_r \pm \varepsilon^*(Q_r) \quad (5)$$

Where  $\alpha$  in a 95% confidence level for the standard normal distribution is set 1.96 and  $\varepsilon^*$  is the width between the upper limit and the lower limit in the 95% confidence level.

Tomkins (2012) investigated the uncertainty of the rating curve  $\varepsilon_2(Q_r)$  by considering the deviation of observed data and estimated discharge derived by the rating curve. Therefore, the relative deviations of measurements from the corresponding rating curve estimations are calculated to obtain the quality of rating curves:

$$D = [(Q_r - Q_e) / Q_e] \times 100 \quad (6)$$

Where  $D$  is the fitting degree between the measured discharge and the predicted discharge from the rating curve for each of the measurements.  $D$  values can be analyzed as Table 1:

**Table 1.** Classification of  $D$  for the analysis of the results of the estimate of the rating curve.

Good	$D = \pm 10\%$
Acceptable	$D = \pm 11 - 20\%$
Suspect	$D = \pm 21 - 50\%$
Poor	$D \geq \pm 50\%$

## 3. Results and discussion

### 3.1. The effect of roughness on the rating curve

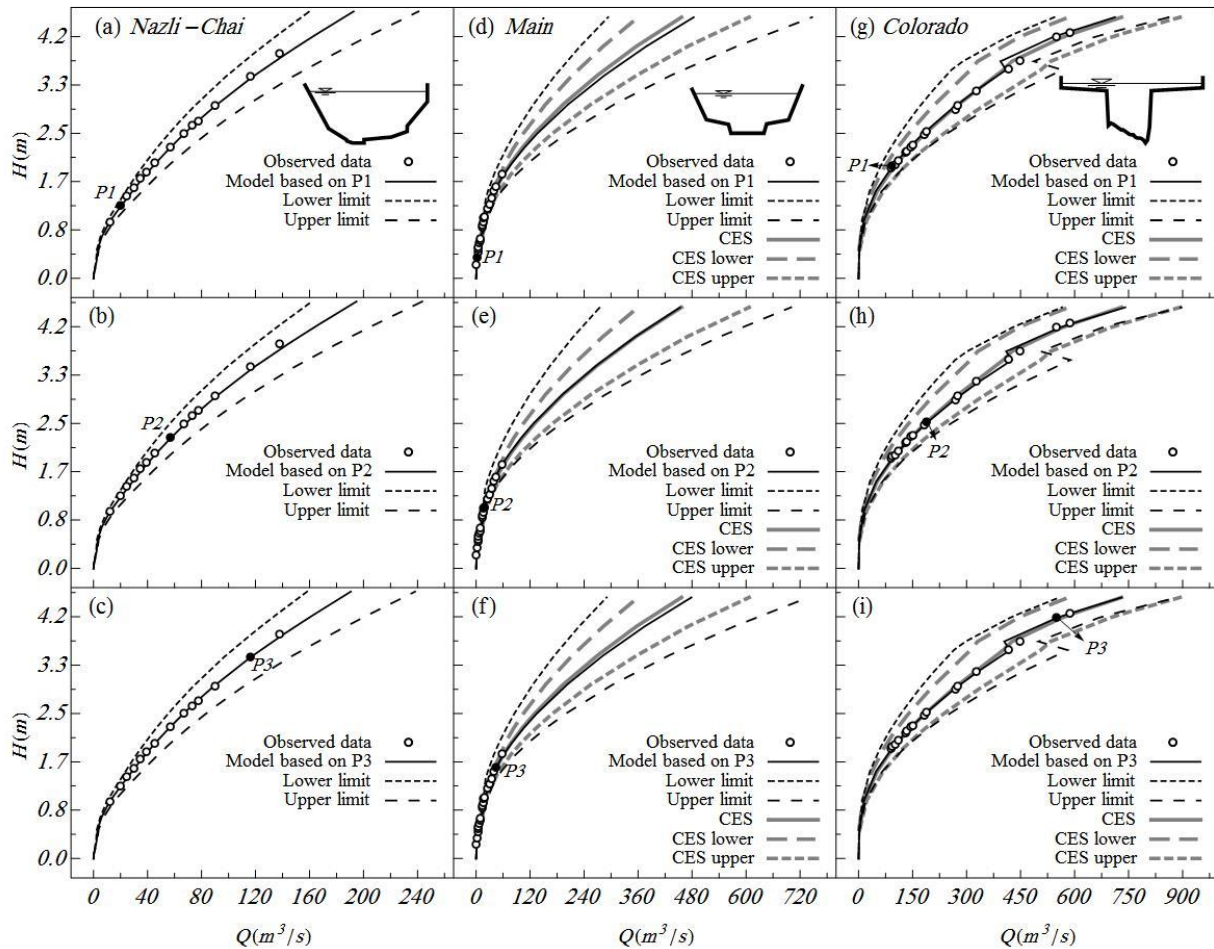
In order to determine the effect of roughness on the rating curve, roughness coefficients are usually defined as an interval. For this reason, the effect of vegetation changes in different conditions is considered in variable roughness coefficients and with the definition of maximum and minimum values. In Fig. 1, in addition to the estimated rating curve by the proposed method, based on observed data P1, P2 and P3, the limits of maximum and minimum uncertainty due to roughness uncertainty are plotted. As can be seen in Figs. 1e and 1h, the results of the two methods are more consistent by the selection of the P2 as a reference level in the proposed method. Also, in most cases, the uncertainty of the proposed method is higher than the CES method. Moreover, there is no significant difference in the results based on different reference levels in the Nazli-chai River (Figs. 1a-1c), which may be due to the less broad cross-section of the river compared to the other two rivers.

### 3.2. Global uncertainty

In the following, Eq. 5 is used to calculate the global uncertainty. As can be seen, in the Nazli-chai River, the maximum  $\varepsilon^*$  values reach approximately 30%, which is based on the P2 reference level for estimation of the rating curve. For comparison of the studied rivers, Table 2 presents the average values of  $\varepsilon^*$  based on the P1, P2 and P3. The mean global uncertainty of the proposed rating curve based on three reference observation data in Nazli-chai, Main and Colorado Rivers are estimated to be 24.3, 33.1 and 42.5%, respectively. Also, by using Tomkins's proposed method (Eq. 6), the quality of the rating curve estimation is classified as "good" and "acceptable", which indicates the favorable quality of the estimations when roughness uncertainty is not considered. However, the quality of the estimates is reduced when the roughness uncertainty is considered.

### 4. Conclusions

The stage-discharge relationship is one of the essential inputs in hydraulic and hydrological models that can be used for flood control and water resource management. The geometric and hydraulic information of the river cross-sections are required to obtain the rating curve relationship. In addition to the roughness parameter, various factors such as the extrapolation of the rating curve and the direct measurement error of the discharge can be effective in estimating the results. In this research, the basis of the estimation of the rating curves is the concept of isovel contours SPM method. Observed data in the Nazli-chai River in Iran, Main River in England and Colorado River in Argentina are used to investigate the global uncertainty in the rating curve estimation.



**Fig. 1.** Estimated rating curves and uncertainty analysis of roughness variation based on different reference point P1, P2 and P3 in the (a-c) Nazli-chai, (d-f) Main and (g-i) Colorado Rivers and Comparison with CES Method.

**Table 2.** Average values of  $\epsilon^*$  based on different reference levels (P1, P2 and P3) in the three studied Rivers.

Reference level	The average values of $\epsilon^*$		
	Nazli-chai	Main	Colorado
P1	23.86	32.1	45.9
P2	24.4	35	39.8
P3	24.7	32.3	41.7

The results show that if the higher accuracy for the estimation of the roughness coefficient as the input parameter is considered, the uncertainty in the discharge estimation will be reduced and the quality of the measurements will increase. Also, the mean global uncertainty of the proposed stage-discharge relationship based on three reference observation data (P1, P2 and P3) in Nazli-chai, Main and Colorado Rivers are calculated to be 24.3, 33.1 and 42.5%, respectively.

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