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**EXTENDED ABSTRACT**

# Finite Element Modelling of Smart Adaptive Composite Beam

Yaser Shahbazi [a,*], Hamid Reza Mirdamadi [b], Mohammad Reza Chenaghlou [c]

[a] *Faculty of Architecture and Urbanism, Tabriz Islamic Art University, Tabriz 5164736931, Iran*
[b] *Mechanical Engineering Department, Isfahan University of Technology, Isfahan 8415683111, Iran*
[c] *Civil Engineering Department, Sahand University of Technology, Tabriz 53318-11111, Iran*

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## 1. Introduction

In the present paper, electromechanical finite element modeling of a smart adaptive composite beam is presented. The model is formulated based on linear electromechanics and electro kinematics assumptions. The proposed model is a three layers piezoelectric composite beam that acts as a transverse actuator. The elastic material of the core is isotropic whereas the outer piezoelectric layers are orthotropic. The accuracy of analytic and numerical models is demonstrated by examining the simulation of the two principles of mechanical and electrical energy conservation in a finite element program and also comparing its results with the ANSYS numerical model. In the numerical simulation of the finite element model, there are three mesh including 10, 50, and 100 elements. The parametric simulation consists of three mechanical, electrical, and electromechanical static loading sets. By comparing the results of modeling in the finite element programming and ANSYS, and verifying the principle of electromechanical energy conservation, it can be concluded that the proposed finite element model is efficient and accurate.

## 2. Methodology

### *2.1. Finite element Formulation*

The smart composite beam model presented in this research is based on linear electromechanical and electro kinematic hypotheses (Cotoni et al., 2006; Gornandt A and Gabbert U, 2002). This means that the electric and mechanical field variables are so small that the theories of elasticity, piezoelectricity, and linear dielectrics can be applied. Elastic materials (core layer) are isotropic but piezoelectric materials (outer layers) are orthotropic. The transverse displacement of all layers at any given cross-sectional area of the longitudinal axis of the beam is equal to the thickness of the beam. The outer layers have assumed the Euler-Bernoulli beam and the elastic core of the beam follows Timoshenko's beam hypotheses. The normal stress (perpendicular to the upper and lower sides of the beam) is negligible. Finally, the piezoelectric layers are polarized in the direction of thickness and parallel to the direction of the applied electric field intensity, and both of them are perpendicular to the neutral axis of the smart beam, to establish the state of the axial actuation mechanism. The kinetic energy functions KE, electromechanical H, the work of mechanical and electrical external forces W, the Lagrangian function L, and the Hamilton generalized variational principle for this continuous piezoelectric environment are defined as follows:

* Corresponding Author
*E-mail addresses:* y.shahbazi@tabriziau.ac.ir (Yaser Shahbazi), hrmirdamadi@cc.iut.ac.ir (Hamid Reza Mirdamadi), mrchenaghlou@sut.ac.ir (Mohammad Reza Chenaghlou).

$$KE = \int_V \tfrac{1}{2}\rho \dot{u}_i \dot{u}_i dV \ \ , \ H = \int_V \tfrac{1}{2}(\sigma_{ij}\varepsilon_{ij} - D_i E_i) dV \ , \ W = \int_S (\bar{T}_k \delta u_k - \bar{Q}\delta\varphi) \ , \ L = \int_V \left[\tfrac{1}{2}\rho \dot{u}_i \dot{u}_j - H_0 E_k\right] dV$$

$$\int_{t_0}^{t_1} \left\{ \delta \int_V \left(\tfrac{1}{2}\rho \dot{u}_i \dot{u}_j dV\right) - \delta \int_V H_0(\varepsilon_{ki}, E_k) dV + \int_S [\bar{T}_k \delta u_k - \bar{Q}\delta\varphi] dV \right\} \tag{1}$$

Based on the degrees of freedom of the whole structure, $q = [u_1' \ \ \tilde{u}_1 \ \ w_1 \ \ w_1' \ \ u_2' \ \ \tilde{u}_2 \ \ w_2 \ \ w_2']^T$ , The linear differential equation of motion of the structure will be $M\ddot{q} + Kq = F_m^e - F_{me}^e$. Where, Finite element stiffness, $[K_e]$ and mass, $[M^e]$, matrices are btained as below:

$$[K^e] = \int_0^L \left\{ \begin{array}{l} c_{11}^{\alpha *} A_\alpha \int_0^{Le} \left(B_{am} B_{am}^T + B_{bm} B_{bm}^T\right) \\ + c_{33}^c A_c \int_0^{Le} \left(B_{cm} B_{cm}^T\right) \\ \bar{c}_{11}^{\alpha} I_a \int_0^{Le} \left(B_{ab} B_{ab}^T + B_{bb} B_{bb}^T\right) \\ + c_{33}^c I_c \int_0^{Le} \left(B_{cb} B_{cb}^T\right) \\ + c_{55}^c A_c \int_0^{Le} \left(B_{cs} B_{cs}^T\right) \end{array} \right\} dx \qquad [\mathrm{M^e}] = \int_0^{\mathrm{L_e}} \left\{ \begin{array}{l} \left(2\rho_\alpha A_\alpha + \rho_c A_c\right) B_{txm} B_{txm}^T \\ + \dfrac{1}{2}\left(\rho_\alpha A_\alpha + \rho_c \dfrac{I_c}{h_c^2}\right) B_{txd} B_{txd}^T \\ + \rho_c \dfrac{I_c}{h_c} \lambda B_{txd} B_r^T + \rho_c \dfrac{I_c}{h_c} \lambda B_r B_{txd}^T \\ + \left(2\rho_\alpha A_\alpha + \rho_c A_c\right) B_{tz} B_{tz}^T \\ + \left(2\rho_\alpha A_\alpha + \rho_c A_c \lambda^2\right) B_r B_r^T \end{array} \right\} dx \tag{2}$$

Then, by writing a numerical algorithm in MATLAB software, the results of the analyzes performed in the finite element model for mechanical, electrical and electromechanical loads as input and mechanical displacement as output are presented. In mechanical analysis, a static external load of 0 to 2N is applied to the end of the beam. In electrical charge, however, a potential difference of 0 to 100V is created on each of the piezoelectric actuators. The deformation created under mechanical charge and electrical charge are opposite to each other. In other words, the displacement caused by electric voltage is a compensatory response of piezoelectric stimuli to undesirable external static loads. Finally, the simulated behavior of the static performance of a piezoelectric actuator in electromechanical loading is investigated. In this way, static loads are applied to the end of the beam as before, and at the same time, a potential difference is applied to each of the piezoelectric actuators to eliminate or reduce the displacement of the end of the smart composite beam. Load-displacement relationships and the difference between internal and external energies for the 10, 50 and 100 meshes of different analyzes are shown in Fig. 1 to 3, respectively. The diagrams show and validate the principle of electrodynamic energy conservation in the finite element model.

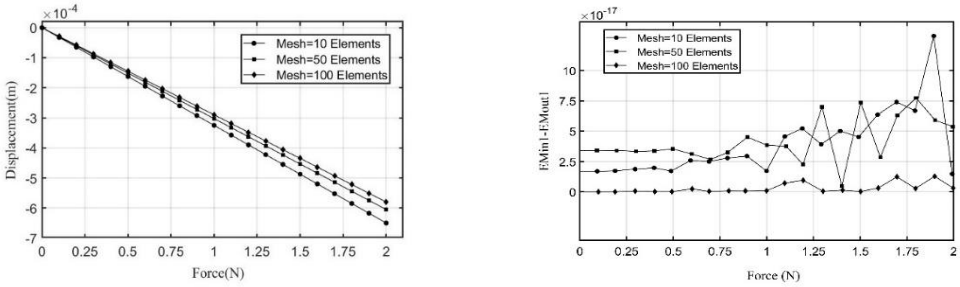


**Fig. 1.** Load-displacement and the principle of electromechanical energy conservation

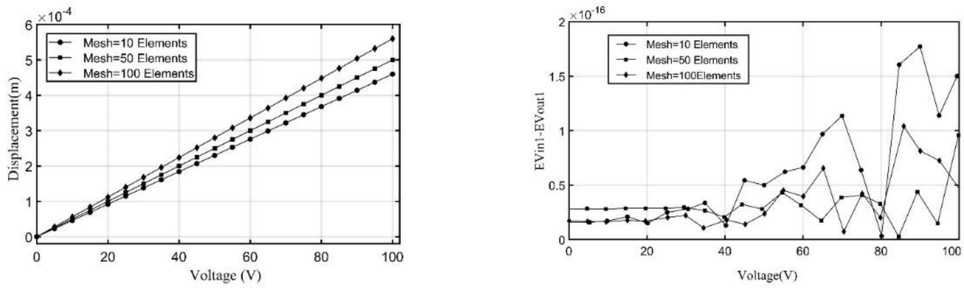


**Fig. 2.** Voltage- displacement and the principle of electromechanical energy conservation

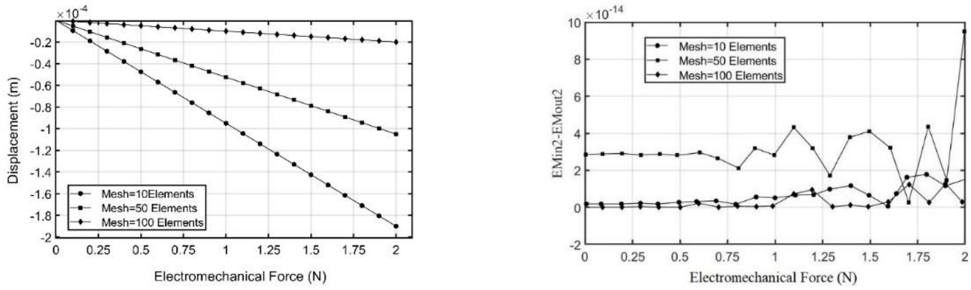


**Fig. 3.** Electromechanical-displacement and the principle of electromechanical energy conservation

## *2.2. Ansys modeling and verification*

The FEM-based software package, ANSYS, was used for the numerical modeling and verification of the finite element model presented in the previous section. When assigning an element to piezoelectric layers, the most appropriate element compatible with the properties of the piezoelectric material in the field of three-dimensional coupling problems for the production of smart composite beams should be used. In this software, piezoelectric elements SOLID5 or SOLID226 can be used. In addition to movement degrees of freedom, the piezoelectric elements have degrees of freedom of electrical potential required for electromechanical interaction problems. Also, the host structure requires degrees of freedom compatible with the degrees of freedom of the piezoelectric actuators in the connection areas. The best element for the elastic layers will be the SOLID45 element. Using the SOLID45 element, to model the middle core type, it is possible that normal and shear stresses should be considered in the host part of the smart composite structure. Mechanical, Electrical, and electromechanical static analyses were done with 50 element meshes to verify the finite element formulations which are presented in the previous section (Fig. 4 to Fig. 6).

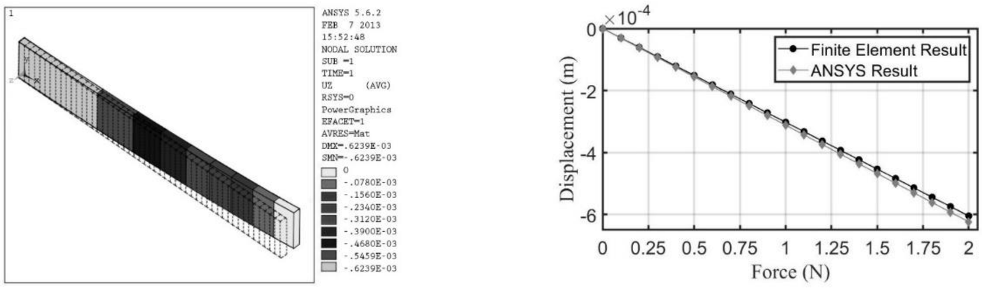


**Fig. 4.** Comparison of proposed finite element model and ANSYS: load-displacement

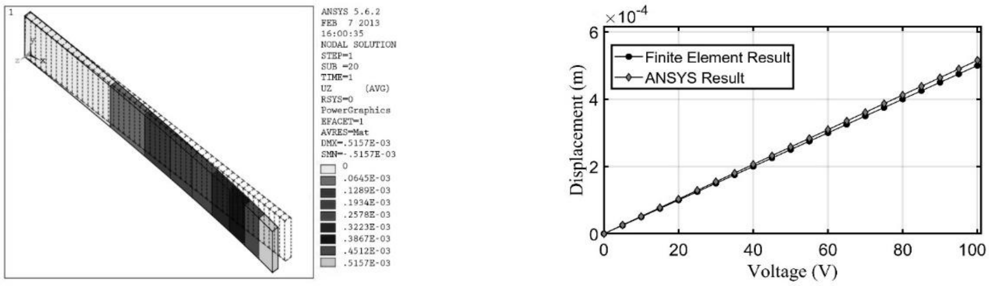


**Fig. 5.** Comparison of proposed finite element model and ANSYS: voltage-displacement

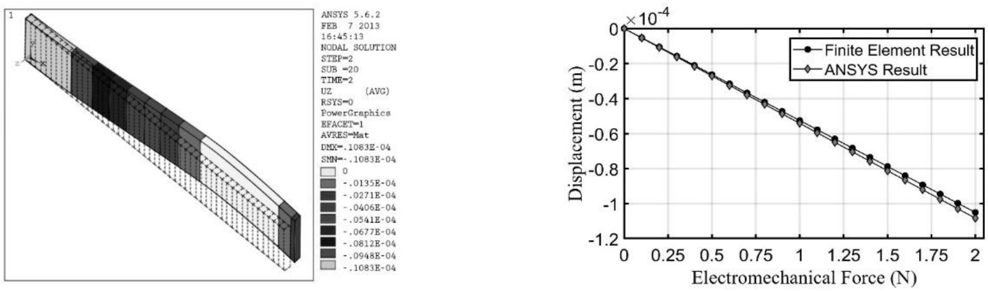


**Fig. 6.** Comparison of proposed finite element model and ANSYS: electromechanical-displacement

## 3. Conclusions

In this paper, a finite element numerical formulation of a smart composite orthotropic beam is obtained based on the theory of continuous motion kinetics. In this formulation, the second-order partial differential equation system of motion was developed using Hamiltonian's advanced mechanics. The direction of polarization and the intensity of the electric field applied to the active material are parallel to the thickness of the beam. In the numerical simulation in the finite element model, three 10, 50 and 100 elements are created. Then, in a parametric analysis, three sets of mechanical, electrical, and electromechanical static loading were performed. Two criteria were selected to validate the finite element model and the written program. The first criterion is to estimate the principle of electromechanical energy conservation between the electro elastic and internal kinetic potential energies with the external work performed by the electromechanical forces. The second criterion is the comparison between the displacements resulting from the numerical model and the ANSYS results. The results show that the finite element formulation has good accuracy and efficiency.

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