



Probability Distribution of Near-Bed Water Particle Velocity Under Wave Action on a Sandy Beach

Seyed Ahmad Neshaei ^{a,*}, Arvin Jalali Gilani ^a, Mir Abdolhamid Mehrdad ^a, Ata Ghabraei ^a, Pooya Rezadoost ^b

^a Department of Civil Engineering, Faculty of Engineering, University of Guilan, Rasht, Iran

^b Faculty of Civil Engineering, K.N. Toosi University of Technology, Tehran, Iran

Received: 12 November 2025; **Received in revised form:** 10 April 2026; **Accepted:** 28 July 2026

ABSTRACT:

This study investigates the most suitable probability distribution function for near-bed water particle velocity under wave action on a sandy beach. Initially, an experimental study was conducted to measure near-bed particle velocities under irregular wave conditions. Based on these measurements, six different probability distribution functions were proposed to characterize particle velocity. Four of these distributions were derived by integrating the probability distribution of wave height with two wave theories: long wave theory and second-order Stokes theory. The remaining two distributions were developed by combining the probability distribution of sea surface displacement with the same wave theories. A detailed comparison between the proposed distributions and the experimental data indicated that the probability distribution based on sea surface displacement, combined with second-order Stokes theory, provides the highest accuracy. This distribution is particularly valuable for practical engineering applications, as it can be derived from readily available sea surface displacement data, which can be easily obtained using standard buoy measurements.

KEYWORDS:

Probability distribution, Near-bed water particle velocity, Sandy beach, Second-order stokes theory, Sea surface displacement.

1. Introduction

Near-bed water particle velocity plays a crucial role in coastal morphology studies, as intense storm events generate large orbital velocities that substantially increase sediment concentration in this region (Dong and Zhang, 2002; Khan-Mozahedy et al., 2015). Grossmann et al. (2022) emphasized the significant influence of near-bed water particle velocity on sediment transport, particularly in current-related transport, which occurs over larger vertical extents and drives offshore bar migration. Liu et al. (2014) investigated the role of near-bed water particle velocity, influenced by tides and

wind, in sediment transport within estuaries, highlighting its importance in understanding and predicting sediment dynamics in these complex environments. Auel et al. (2017) examined the influence of near-bed water particle velocity on sediment transport, including its effects on particle impact velocity, impact angle, and the transfer of horizontal energy to the bed. In addition to its role in coastal morphology, accurate prediction of the near-bed velocity waveform responsible for generating the maximum wave load on seabed structures is essential for evaluating their stability in coastal environments (Kawamata and Kobayashi, 2023).

Elfrink et al. (2006) proposed empirical formulas that characterize near-bed orbital velocities generated by irregular waves in shallow water. Pu (2021) proposed a novel approach for improving velocity measurements near the bed by introducing a bed realignment method and investigating its effects on sediment transport. According to Sultan (1992), the Gram-Charlier distribution proved less effective than the Gaussian distribution for fitting the measured instantaneous orbital velocities. In contrast, the Beta-Rayleigh distribution demonstrated a closer agreement with measured orbital velocity amplitudes than the traditional Rayleigh distribution. Song and Wu (2000) found that, under the second-order random wave approximation, vertical orbital velocities follow a Gaussian distribution, whereas horizontal orbital velocities deviate from this distribution when the water depth is less than 10 meters. You (2009) reported that the Rayleigh distribution tends to overestimate large orbital velocities U_q (where U_q is the average of the n largest orbital velocities from m wave orbital velocity amplitudes) when the parameter Q (defined as the ratio of the number of largest orbital velocities to the total number of wave orbital velocity amplitudes) is less than 0.1.

Recent studies have further highlighted the importance of understanding near-bed velocity distributions in various contexts. Li et al. (2024) developed a semi-empirical model for near-bed velocity and turbulence distribution in scour holes downstream of submerged structures, demonstrating that probabilistic frameworks can accurately represent near-bed velocity profiles in complex hydraulic environments. Similarly, Yu et al. (2025) showed that continuous near-bed movements of microplastics in flumes follow normal distributions, reinforcing the applicability of probabilistic approaches in modeling near-bed particle transport. Zhao et al. (2024) emphasized that near-bed flow structures and turbulence strongly influence particle transport and bedload intensity, and demonstrated that logistic regression can effectively estimate near-bed particle velocities in turbulent flows. Field observations by Zhou et al. (2025) revealed multi-year variations in near-bed flow velocity and suspended sediment in estuarine environments, confirming the importance of accurately characterizing near-bed velocity for predicting sediment dynamics. Furthermore, Khadami et al. (2025) analyzed near-bed velocity and bed shear stress in mangrove-vegetated estuaries, showing that near-bed flow patterns, even under tidal influences, directly affect particle transport and resuspension.

The primary objective of this study is to develop a practical framework for constructing the probability distribution of near-bed water particle velocity under wave action on a sandy beach—a

process that can be implemented without the need for specialized equipment or extensive data. To achieve this, a laboratory study was conducted to directly measure near-bed velocities. Based on these measurements, six different probability distributions were developed to characterize near-bed particle velocity, derived from the probability distributions of wave height and sea surface displacement. In this process, two wave theories—the long wave theory and second-order Stokes theory—were employed. Finally, a comparative analysis was performed between the proposed distributions and experimental measurements of near-bed particle velocity, and the distribution that showed the closest agreement with the experimental data was identified as the most accurate representation.

Despite the extensive body of research on near-bed particle velocity and its role in sediment transport, a key limitation remains: the lack of a practical and transferable probabilistic framework capable of estimating near-bed velocity distributions without relying on direct velocity measurements. Many existing approaches are either based on empirical relationships developed under specific conditions or require detailed in-situ velocity data, which are often difficult and costly to obtain. Furthermore, limited attention has been given to establishing a direct and systematic linkage between easily measurable wave-related parameters such as sea surface displacement and the probabilistic characterization of near-bed particle velocity within a unified theoretical framework. Therefore, the objective of the present study is to develop a physically consistent and practically applicable methodology for deriving the probability distribution of near-bed water particle velocity using accessible hydrodynamic data. This is achieved by integrating probability distributions of wave characteristics with wave theories and evaluating multiple formulations within a unified framework to identify the most reliable representation.

2. Experimental set-up

To determine the near-bed water particle velocity, laboratory experiments were conducted in a large-scale two-dimensional wave flume with dimensions of 60 m in length, 2.80 m in width, and 1.50 m in depth (Fig. 1). To optimize the use of sediment material, only one-third of the flume width was covered with sand, forming the active bed region.

At the downstream end of the flume, a wave-dissipation beach was constructed based on an equilibrium profile supported by a rigid metal framework, following the equation:

$$h = Ax^{\frac{2}{3}} \quad (1)$$

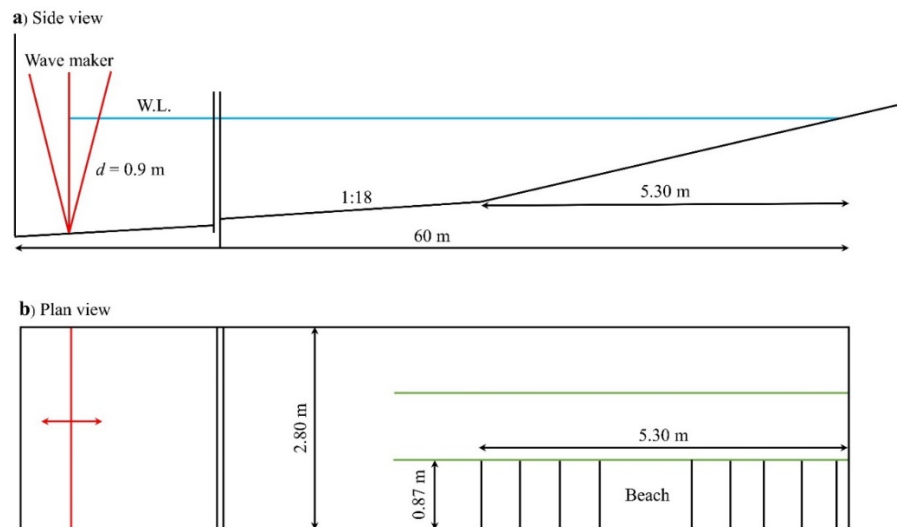


Fig. 1. Illustration of experimental set-up: **a)** Side view, **b)** Plan view

In this equation, x and h denote the horizontal and vertical coordinates measured from the shoreline, respectively, with h measured downward from the still water level. The parameter A depends on sediment characteristics. The beach surface was covered with sand, resulting in a hydraulically rough granular surface (gravel-type), where energy dissipation occurs through bed friction and sediment permeability. No artificial roughness elements such as cubes or engineered units were used.

The constructed beach profile had a total length of 5.30m, starting from the shoreline and transitioning to a uniform slope of 1:18 toward the deeper region ($h=0.9\text{m}$). This configuration leads to a gradual change in slope from 1:18 at $x=5.30\text{m}$ to 1:5.5 at the shoreline. Additionally, the profile extended 2.50m landward of the shoreline with a slope of 1:10. This combined geometric and material configuration enhances wave energy dissipation through both slope-induced breaking and frictional losses, thereby reducing wave reflection within the flume.

Irregular waves were generated at the upstream end of the flume using an electro-hydraulic wave maker controlled by a personal computer and driven by a JONSWAP spectrum. This spectrum is widely used to represent developing and storm-dominated sea states rather than long-term calm conditions. Therefore, the present experiments correspond to energetic wave conditions representative of storm environments. The main spectral characteristics of the generated waves were a significant wave height of $H_s=80\text{mm}$, a significant wave period of $T_s=1\text{ sec}$, a deep-water wavelength of $L_0=1560\text{mm}$, and a wave steepness of $S_0=0.051$.

The wave time series were controlled and monitored through the wave maker input signal, which defines the target JONSWAP spectrum. These signals were used to ensure that the generated wave

field satisfied the prescribed spectral characteristics.

Near-bed water particle velocities were measured using a one-component fiber-optic Laser-Doppler Anemometer (LDA) manufactured by Dantec Electronics. The LDA system operated based on a backscatter technique using dual laser beams. The measurement system included a 105mm long probe with a focal length of 50mm and a beam spacing of 8 mm. Additionally, a frequency tracker and shifter were used for accurate detection of the velocity component. The Doppler frequency ranged from 0.1 to 1 MHz, with a frequency shift of 500kHz.

Velocity measurements were conducted at a fixed location 1200 mm from the shoreline and at a vertical distance of 15mm above the bed. The local water depth at this location was 143.8mm. Based on the linear dispersion relationship, the corresponding wavelength was calculated as 1072.12mm.

To minimize the influence of reflected waves, the sloping beach acted as a passive absorber, and the measurement point was selected sufficiently far from the boundary to reduce reflection effects. Furthermore, only the steady-state portion of the recorded time series-after the decay of initial transients-was used for analysis. The duration of data acquisition was selected to ensure statistical convergence and reliable estimation of velocity distributions.

To verify the applicability of the second-order Stokes wave theory at the measurement location, the criterion proposed by Dean and Dalrymple (1991) was employed. According to this criterion, the theory is valid when the parameter L^2H/d^3 is smaller than a specified limiting value. For the present wave conditions, this parameter was calculated as 19.36, which is significantly smaller than the limiting value $64\pi^2/3$. This confirms that the wave conditions fall within the valid range of the second-order Stokes theory.

3. Preliminary and essential definitions

3.1. Probability distribution of wave height

The Rayleigh distribution is widely used to describe the statistical distribution of wave height, as expressed in Eq. (2), where H denotes wave height and H_{rms} is the root-mean-square wave height. From a physical perspective, this distribution is applicable for narrow-banded, random wave fields where wave energy is concentrated around a dominant frequency.

However, a limitation of the Rayleigh distribution is that it does not explicitly incorporate the effect of water depth on wave statistics. To overcome this limitation, a modified Rayleigh distribution has been proposed by Massel (1996), as given in Eq. (3). In this formulation, a depth-related parameter $n = \bar{H}/d$ is introduced, which accounts for changes in wave statistics under varying water depths. This modification is particularly important in shallow water conditions, where depth-induced wave transformation becomes significant.

$$f_1(H) = \frac{2H}{H_{rms}^2} \exp\left[-\left(\frac{H}{H_{rms}}\right)^2\right] \quad (2)$$

$$f_2(H) = \frac{b'}{\bar{H}} \left(\frac{H}{\bar{H}}\right)^{\frac{1+n}{1-n}} \exp\left[-a' \left(\frac{H}{\bar{H}}\right)^{\frac{2}{1-n}}\right];$$

$$n = \frac{\bar{H}}{d}; a' = \frac{\pi}{4(1+\frac{n}{\sqrt{2\pi}})}; b' = \frac{2a'}{1-n} \quad (3)$$

It is important to emphasize that in this study, the Rayleigh distribution is not used as a deterministic description of wave motion or as a direct representation of near-bed velocity. Instead, it is employed as a statistical representation of wave height variability under random sea states.

In physical terms, wave height is a strictly non-negative variable and is therefore suitable for Rayleigh-type modeling. However, near-bed water particle velocity exhibits oscillatory behavior, including both positive and negative values due to wave-induced orbital motion. This fundamental difference implies that wave height statistics cannot directly represent velocity statistics.

Therefore, the Rayleigh-based formulation in this study should be interpreted as an intermediate stochastic descriptor of the sea state, which is later transformed into velocity statistics using deterministic wave theories. This interpretation motivates the use of alternative surface-displacement-based formulations, which naturally preserve the oscillatory nature of fluid particle motion.

3.2. Probability distribution of sea surface displacements

Sea surface displacement $\eta(t)$ is commonly modeled using a Gaussian probability density function due to the central limit behavior of multiple superimposed wave components. From a physical standpoint, sea surface elevation results from the linear superposition of many independent wave components with random phases. According to the central limit theorem, this leads to an approximately Gaussian distribution for surface displacement.

The Gaussian distribution is therefore used in this study to represent the symmetric fluctuations of the free surface around its mean value. The displacement is characterized by its mean $\bar{\eta}$ and variance S_η , and its probability density function is given by Eq. (4).

$$f(\eta) = \frac{1}{\sqrt{2\pi S_\eta}} \exp\left[-\frac{(\eta - \bar{\eta})^2}{2S_\eta}\right] \quad (4)$$

Unlike wave height, surface displacement directly contains both positive and negative values, making it more suitable for describing oscillatory wave motion and near-bed particle dynamics.

3.3. Wave theories

3.3.1. Long wave theory

Eq. (5) and Eq. (6) describe long-wave behavior under shallow-water conditions, where the wavelength is much larger than the water depth.

Eq. (5) defines the surface profile η , while Eq. (6) describes the horizontal water particle velocity u . Here, C is the wave celerity, σ is the angular frequency, k is the wave number, and ϕ is the phase angle, taking values π and 2π for negative and positive velocities, respectively.

From a physical point of view, long wave theory assumes that the horizontal particle velocity is nearly uniform over depth due to the shallow-water constraint. The vertical velocity component is comparatively small and varies approximately linearly from the bed to the free surface.

This behavior reflects the dominance of hydrostatic pressure variations and weak vertical structure in shallow-water wave motion, as described by Dean and Dalrymple (1991).

$$\eta = \frac{H}{2} \cos(kx - \sigma t) = \frac{H}{2} \cos \phi \quad (5)$$

$$u = \frac{gkH}{2\sigma} \cos \phi = \frac{\eta C}{d} \quad (6)$$

3.3.2. Second-order Stokes theory

The second-order Stokes wave theory is applicable to intermediate water depths and weakly

nonlinear wave conditions. Unlike linear theory, it includes second-order corrections that account for nonlinear wave deformation. Eq. (7) defines the relationship between surface elevation and wave height, while Eq. (8) relates wave height to particle velocity. From a physical perspective, second-order Stokes theory captures key nonlinear effects such as wave crest sharpening and trough flattening, while still assuming periodic and irrotational flow conditions.

$$\eta = \frac{k \cosh(kd)}{16 \sinh^3(kd)} [2 + \cosh(2kd)] \cos(2\phi) H^2 + \dots + \frac{\cos \phi}{2} H \quad (7)$$

$$u = \frac{3\sigma k \cosh[2k(d+z)] \cos 2\phi}{16 \sinh^4(kd)} H^2 + \dots + \frac{gk \cosh[k(d+z)] \cos \phi}{2\sigma \cosh(kd)} H \quad (8)$$

However, this theory does not provide a direct explicit relationship between surface elevation and particle velocity. Therefore, an intermediate transformation procedure is required:

- **Step 1** Solve Eq. (7) to express wave height as a function of surface elevation (Eq. (9)).
- **Step 2** Solve Eq. (8) to express wave height as a function of velocity (Eq. (10)).
- **Step 3** Eliminate wave height to obtain a direct relation between surface elevation and velocity (Eq. (11)).

This procedure does not introduce new physics, but provides a consistent kinematic transformation linking free-surface motion to near-bed velocity under second-order wave assumptions.

4. Deriving the probability distribution function of near-bed water particle velocity

This study develops the probability distribution of near-bed water particle velocity in the surf zone using two complementary stochastic frameworks. From a physical-statistical viewpoint, wave height and sea surface displacement are considered primary random variables describing the sea state. In contrast, near-bed water particle velocity is treated as a derived variable obtained through deterministic wave-theory-based transformations.

Mathematically, this transformation is performed using the change-of-variables method. For a general transformation $Y=h(X)$, the probability density function of Y is obtained using Eq. (12), where J represents the Jacobian of the

transformation. From a physical interpretation perspective, the Jacobian does not introduce new physics; rather, it ensures conservation of probability during the transformation of variables.

Based on this framework, six probability distributions of near-bed velocity are obtained by combining:

- wave height statistics (Rayleigh and modified Rayleigh),
- sea surface displacement statistics (Gaussian),
- and two wave theories (long wave theory and second-order Stokes theory).

This is illustrated in Fig. 2.

$$\left\{ \begin{array}{l} H_1 = \frac{-b_1 \pm \sqrt{\Delta_1}}{2a_1}; \\ \Delta_1 = b_1^2 - 4a_1c_1 \\ c_1 = -\eta \\ b_1 = \frac{\cos \phi}{2} \\ a_1 = \frac{k \cosh(kd)}{16 \sinh^3(kd)} [2 + \cosh(2kd)] \cos(2\phi) \end{array} \right. \quad (9)$$

$$\left\{ \begin{array}{l} H_2 = \frac{-b_2 \pm \sqrt{\Delta_2}}{2a_2}; \\ \Delta_2 = b_2^2 - 4a_2c_2 \\ c_2 = -u \\ b_2 = \frac{gk \cosh[k(d+z)] \cos \phi}{2\sigma \cosh(kd)} \\ a_2 = \frac{3\sigma k \cosh[2k(d+z)] \cos 2\phi}{16 \sinh^4(kd)} \end{array} \right. \quad (10)$$

$$H_1 = H_2 \rightarrow \eta = a_1 \left(\frac{b_1}{2a_1} - \frac{b_2}{2a_2} + \frac{\sqrt{\Delta_2}}{2a_2} \right)^2 - \frac{b_1^2}{4a_1} \quad (11)$$

$$g(y) = f(h^{-1}(y)) \times |J| = f(h^{-1}(y)) \times \left| \frac{dx}{dy} \right| \quad (12)$$

4.1. Velocity distribution derived from wave height statistics

In this approach, wave height is treated as the primary stochastic descriptor of the sea state, while near-bed velocity is derived as a dependent variable. Eq. (6) provides the kinematic relationship between wave height and velocity under shallow-water assumptions. This relation is inverted in Eq. (13), and the Jacobian is computed in Eq. (14). By substituting the Rayleigh distribution into this transformation, the velocity distribution is obtained in Eq. (15).

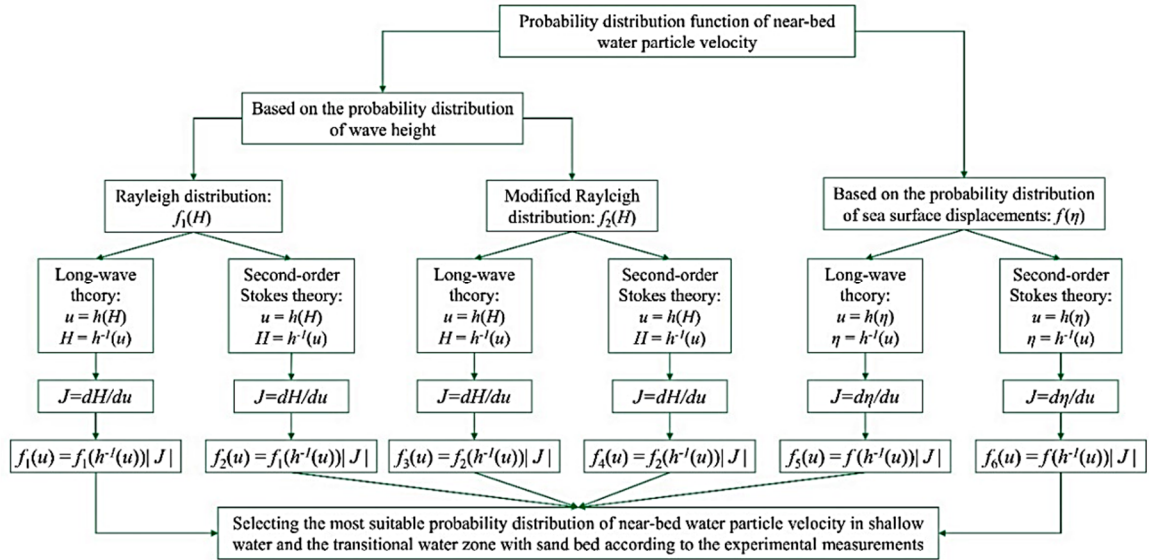


Fig. 2. Procedure for generating six probability distributions for near-bed water particle velocity

Using the modified Rayleigh distribution, Eq. (16) is derived. From a physical standpoint, this formulation assumes that variability in near-bed velocity is entirely driven by variability in wave height under depth-uniform velocity conditions.

In the second-order Stokes framework, Eq. (8) introduces a nonlinear relationship between wave height and velocity. Since this relationship is quadratic, Eq. (17) is used to obtain wave height, and Eq. (18) defines the corresponding Jacobian. By applying Rayleigh-based statistics, the velocity distributions are obtained in Eq. (19) and Eq. (20). Physically, this approach accounts for weak nonlinear wave effects in the transformation from wave statistics to velocity statistics.

$$H = \frac{2\sigma u}{gk \cos \phi} \quad (13)$$

$$J = \frac{dH}{du} = \frac{2\sigma}{gk \cos \phi} \quad (14)$$

$$f_1(u) = \frac{4\sigma u}{H_{rms}^2 gk \cos \phi} \times \dots \times \exp \left[- \left(\frac{2\sigma u}{H_{rms} gk \cos \phi} \right)^2 \right] \left| \frac{2\sigma}{gk \cos \phi} \right| \quad (15)$$

$$f_3(u) = \frac{b'}{H} \left(\frac{2\sigma u}{H gk \cos \phi} \right)^{\frac{1+n}{1-n}} \times \dots \times \exp \left[-a' \left(\frac{2\sigma u}{H gk \cos \phi} \right)^{\frac{2}{1-n}} \right] \left| \frac{2\sigma}{gk \cos \phi} \right| \quad (16)$$

$$H = \frac{-b \pm \sqrt{\Delta}}{2a}; \quad \begin{cases} \Delta = b^2 - 4ac \\ c = -u, \\ b = \frac{gk \cosh[k(d+z)] \cos \phi}{2\sigma \cosh(kd)} \\ a = \frac{3\sigma k \cosh[2k(d+z)] \cos 2\phi}{16 \sinh^4(kd)} \end{cases} \quad (17)$$

$$J = \frac{dH}{du} = \frac{1}{2a} \times \frac{4a}{2\sqrt{\Delta}} = \frac{1}{\sqrt{\Delta}} \quad (18)$$

$$f_2(u) = \frac{2(-b+\sqrt{\Delta})}{2aH_{rms}^2} \exp \left[- \left(\frac{-b+\sqrt{\Delta}}{2aH_{rms}} \right)^2 \right] \times \left| \frac{1}{\sqrt{\Delta}} \right| \quad (19)$$

$$f_4(u) = \frac{b'}{H} \left(\frac{-b+\sqrt{\Delta}}{2aH} \right)^{\frac{1+n}{1-n}} \times \dots \times \exp \left[-a' \left(\frac{-b+\sqrt{\Delta}}{2aH} \right)^{\frac{2}{1-n}} \right] \times \left| \frac{1}{\sqrt{\Delta}} \right| \quad (20)$$

4.2. Velocity distribution derived from sea surface displacement statistics

In this framework, sea surface displacement $\eta(t)$ is considered the primary stochastic input.

Eq. (6) is reformulated as Eq. (21), directly expressing velocity as a function of surface elevation. The Jacobian is given in Eq. (22), and the resulting probability density function is obtained in Eq. (23). From a physical perspective, this formulation directly links free-surface fluctuations to near-bed particle motion under shallow-water conditions, where vertical structure effects are limited.

For second-order Stokes waves, no explicit direct relation exists between surface elevation and velocity. Therefore, Eq. (11) is used as the connecting kinematic relation.

By applying the change-of-variables method and assuming Gaussian statistics for surface displacement, the velocity distribution is obtained in Eq. (25), while Eq. (24) defines the Jacobian. This approach provides a physically consistent mapping from nonlinear surface statistics to near-bed velocity statistics under weakly nonlinear wave conditions.

$$\eta = \frac{d}{\sqrt{gd}} u \quad (21)$$

$$J = \frac{d\eta}{du} = \frac{d}{\sqrt{gd}} \quad (22)$$

$$f_5(u) = \frac{1}{\sqrt{2\pi}S_\eta} \exp \left[-\frac{\left(\frac{d}{\sqrt{gd}} u - \bar{\eta} \right)^2}{2S_\eta^2} \right] \times \left| \frac{d}{\sqrt{gd}} \right| \quad (23)$$

$$J = \frac{d\eta}{du} = \left(\frac{b_1}{2a_1} - \frac{b_2}{2a_2} + \frac{\sqrt{\Delta_2}}{2a_2} \right) \frac{2a_1}{\sqrt{\Delta_2}} \quad (24)$$

$$f_6(u) = \frac{1}{\sqrt{2\pi}S_\eta} \left| \left(\frac{b_1}{2a_1} - \frac{b_2}{2a_2} + \frac{\sqrt{\Delta_2}}{2a_2} \right) \frac{2a_1}{\sqrt{\Delta_2}} \right| \times \dots \times \exp \left[-\frac{\left(a_1 \left(\frac{b_1}{2a_1} - \frac{b_2}{2a_2} + \frac{\sqrt{\Delta_2}}{2a_2} \right)^2 \frac{b_1^2}{4a_1} - \bar{\eta} \right)^2}{2S_\eta^2} \right] \quad (25)$$

5. Evaluating the precision of six probability distributions for near-bed water particle velocity

This section focuses on assessing the accuracy of the six proposed probability distributions for near-bed water particle velocity and identifying the most suitable one. To accomplish this, a comparison will be made between these distributions and the experimental results. The probability density function (PDF) of the measured velocities is shown in Fig. 3. The distribution exhibits a nearly symmetric pattern around the mean, reflecting the bidirectional nature of near-bed velocity fluctuations. It should be noted that the comparison with a Gaussian distribution in this figure is presented only as a qualitative reference to illustrate the general shape and symmetry of the measured data.

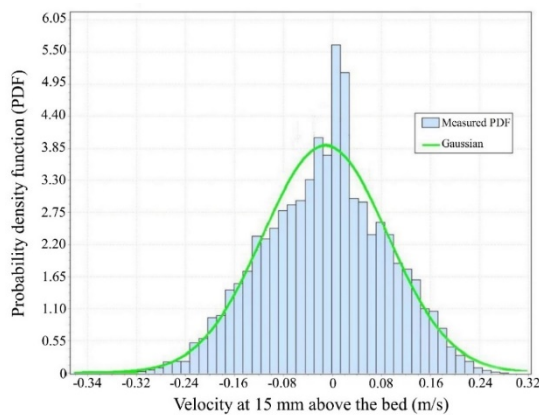


Fig. 3. Comparison of measured PDF with Gaussian probability distribution.

The purpose of Fig. 3 is not to provide a rigorous statistical confirmation of Gaussian behavior. Instead, the accuracy of the proposed probability distributions is evaluated using quantitative criteria, as discussed in the following sections.

Figs. 4 and 5 illustrate the probability distributions of near-bed water particle velocity obtained by integrating wave height probability distributions with long wave theory and second-order Stokes wave theory, respectively. Analysis of both figures reveals that the particle velocity distribution closely follows a Rayleigh distribution, consistent with the statistical properties of wave height distributions. However, when a modified Rayleigh distribution for wave height is employed, the resulting particle velocity distribution tends to approximate a Gaussian shape. In the context of near-bed water particle velocity, which encompasses both positive and negative values due to wave dynamics, the Rayleigh distribution may not adequately capture the full range of velocities. The Rayleigh distribution is inherently suitable only for non-negative variables, as it excludes negative values and focuses solely on the positive domain. Consequently, Eqs. (15), (16), (19), and (20), which rely on the Rayleigh distribution, may not fully represent the observed velocity distribution, especially in scenarios where negative velocities are significant.

Fig. 6 illustrates the probability distributions of particle velocity at a distance of 1200 mm from the beach. These distributions are obtained by combining the probability distribution of sea surface displacements with both the long wave theory and the second-order Stokes theory. The figure demonstrates that employing the distribution of sea surface displacements yields a probability distribution curve for water particle velocities that closely resembles a Gaussian distribution curve. This distribution accurately represents the full range of velocities, including negative values. Fig. 7 further emphasizes the favorable agreement between the second-order Stokes theory and the Gaussian distribution, highlighting their capability to model both positive and negative velocities effectively.

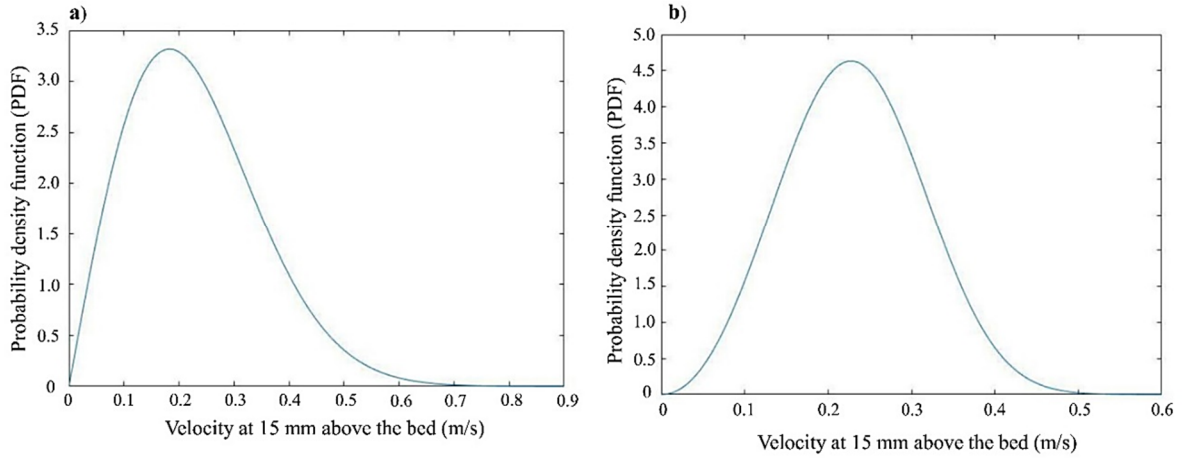


Fig. 4. Probability distribution of near-bed water particle velocity based on long wave theory: **a)** using the Rayleigh distribution, **b)** using the modified Rayleigh distribution.

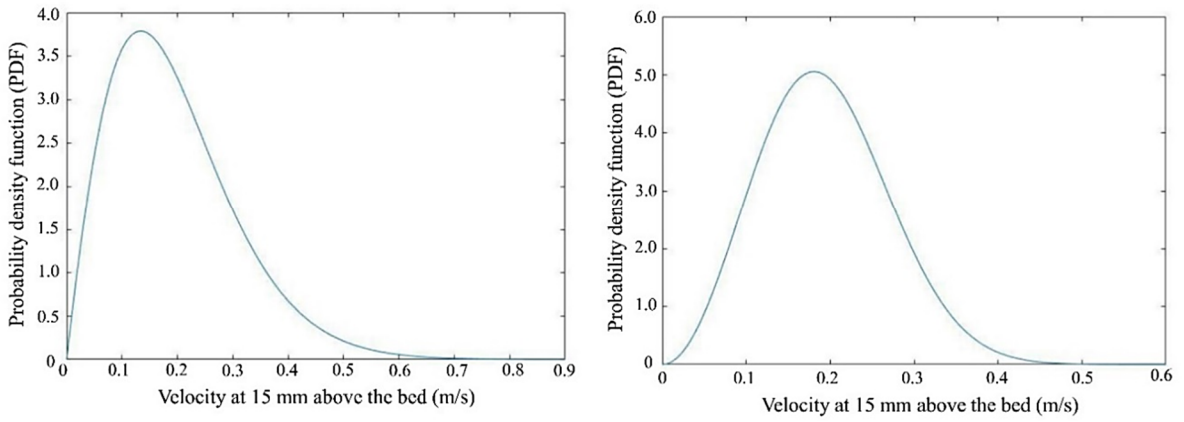


Fig. 5. Probability distribution of near-bed water particle velocity based on second-order Stokes theory: **a)** using the Rayleigh distribution, **b)** using the modified Rayleigh distribution

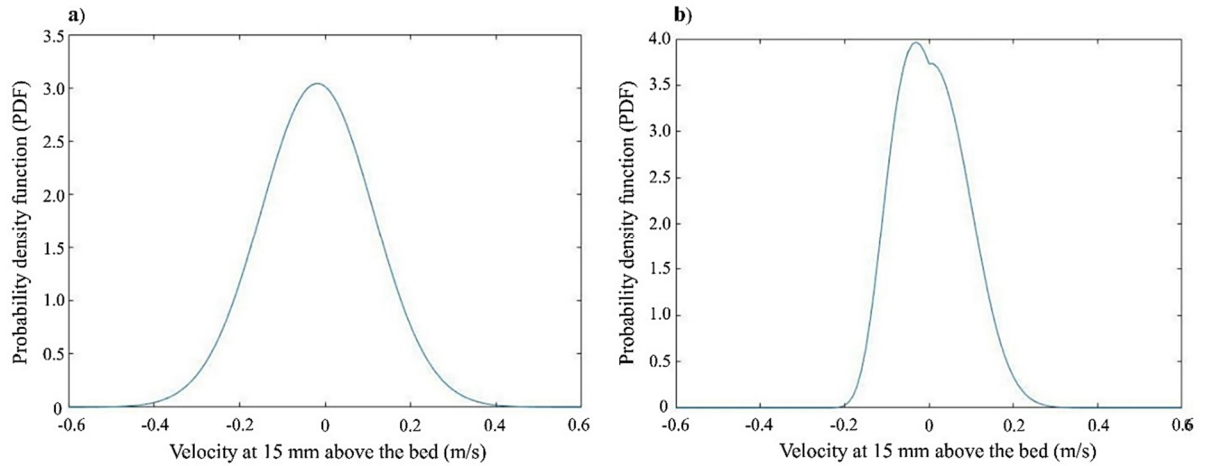


Fig. 6. Probability distribution of near-bed water particle velocity based on the probability distribution of sea surface displacements: **a)** long wave theory, **b)** second-order Stokes theory

To assess the accuracy of the probability distribution of near-bed particle velocity derived from the probability distribution of sea surface displacements, the normalized root mean square error (NRMSE) criterion was employed, defined as:

$$NRMSE = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^n (v_{perd,i} - v_{exp,i})^2}}{V_{min,expmax,exp}} \quad (26)$$

Where $V_{perd,i}$ and $V_{exp,i}$ represent the predicted and experimentally measured velocities at the i -th bin of the probability distribution, respectively, and n is the total number of bins. The normalization was conducted using the range of the experimental velocity values ($V_{min,expmax,exp}$), and the comparison was performed bin-wise on the PDFs rather than on the raw velocity measurements.

This approach ensures that the deviation between the predicted and measured distributions is expressed as a percentage relative to the experimental data range, providing a consistent measure of the model's accuracy across the full spectrum of near-bed velocities.

The calculated error percentages between the experimental results and Eq. (23) and Eq. (25) were found to be 11.56% and 4.59%, respectively. Based on these error percentages, it can be concluded that the distribution derived from the probability distribution of sea surface displacements, in combination with the second-order Stokes theory (Eq. (25)), exhibits an acceptable level of accuracy. The relatively low error percentages suggest that the model provides a reasonably accurate representation of the near-bed particle velocity distribution.

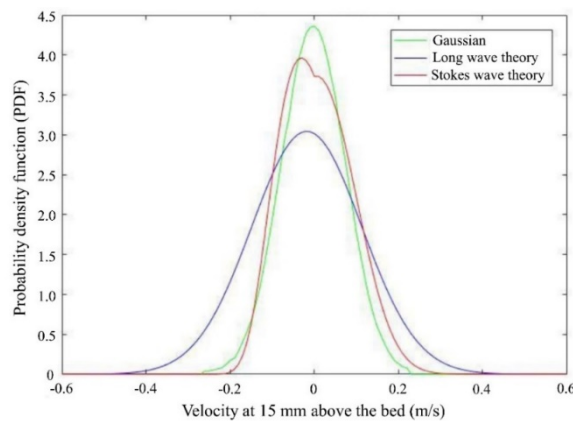


Fig. 7. Comparison of near-bed water particle velocity probability distribution derived from sea surface displacement distribution using long wave theory and second-order Stokes theory with Gaussian distribution.

Table 1. Error percentage between obtained distributions and experimental distribution.

#	Wave theory	NRMSE (%)
1	Long wave	11.56
2	Second-order Stokes theory	4.59

The results of this study provide important insights into the role of the selected wave descriptor in determining the statistical structure of near-bed particle velocity. It is observed that formulations based on sea surface displacement offer a more consistent representation of the bidirectional and oscillatory nature of near-bed velocity compared to those derived from wave height. This is primarily because sea surface displacement inherently preserves directional variability and phase information, which are essential characteristics of near-bed flow

dynamics. In addition, the incorporation of second-order Stokes theory improves the predictive capability of the derived distributions by accounting for nonlinear wave effects, which become increasingly significant in shallow water conditions. From a practical standpoint, the proposed framework allows for the estimation of near-bed velocity distributions using readily available surface measurements, without requiring direct velocity data. Overall, the findings demonstrate that the proposed approach provides a physically interpretable and practically implementable framework, making it suitable for engineering applications where direct measurement of near-bed velocity is not feasible.

6. Limitations and Future Work

This study provides a practical framework for estimating the probability distribution of near-bed water particle velocity based on sea surface displacement. However, several limitations should be acknowledged.

First, the experimental measurements were conducted at a single spatial location, which limits the ability to generalize the results across different cross-shore positions and water depths. As a result, spatial variability in near-bed velocity, particularly under complex hydrodynamic conditions, is not captured.

Second, due to laboratory constraints, the experiments were performed under a single set of wave conditions, and no repeated independent tests were conducted. Nevertheless, the statistical analysis is based on a sufficiently long time series, which ensures a large sample size and reduces the impact of random measurement errors.

Third, the experiments were carried out under controlled wave conditions in a flume, representing primarily shoaling waves. Therefore, important coastal processes such as wave breaking, strong turbulence, and surf zone dynamics are not included in the present analysis.

Finally, although the proposed framework is formulated in a way that can be applied to different water depths using sea surface data, its experimental validation is currently limited to one depth and one measurement point.

Future studies should address these limitations by:

- conducting measurements at multiple cross-shore locations and elevations,
- testing the model under different wave conditions, including storm and breaking waves,
- and validating the approach using field data.

Such extensions would significantly enhance the generality and applicability of the proposed probabilistic framework.

7. Conclusions

In this study, the probability distribution of near-bed water particle velocity under wave action on a sandy beach was investigated. A comprehensive laboratory program was conducted to obtain experimental data on near-bed water particle velocity. Six distinct probability distribution functions were introduced to model the velocity of water particles near the bed.

The results indicated that the probability distribution derived from sea surface displacement data, in combination with second-order Stokes theory, provides the most accurate representation of near-bed particle velocity. A discrepancy of 4.59% was observed when comparing the proposed distribution with the experimental data.

The novelty of this study lies in the use of sea surface displacement data to derive the probability distribution of near-bed water particle velocity. This approach is particularly advantageous because sea surface displacements can be easily measured using buoy-based instruments, making the required data readily accessible. The availability of such data enables efficient estimation of near-bed water particle velocity across various water depth conditions. In addition, the study introduces a unified probabilistic framework that systematically links wave characteristics to near-bed velocity distributions through wave theory, providing a practical alternative to direct velocity measurements.

One limitation of this study is that only a single measurement point was examined due to laboratory constraints. Future studies should extend the analysis to multiple locations in shallow water regions to further validate and refine the proposed probability distribution.

Nomenclature

Greek symbols

$\eta(t)$	Sea surface displacement
$\bar{\eta}$	Mean sea surface elevation
S_{η}	Variance of sea surface displacement
σ	Angular frequency of wave motion
φ	Wave phase angle
π	Phase angle for negative velocity
2π	Phase angle for positive velocity

Wave parameters

H	Wave height
$\bar{H}$	Mean wave height
H_{rms}	Root-mean-square wave height
H_s	Significant wave height
T_s	Significant wave period
L_0	Deep-water wavelength
k	Wave number
d	Water depth

x	Horizontal coordinate
h	Vertical coordinate (measured downward from still water level)
A	Parameter depending on sediment characteristics

Velocity and kinematic variables

u	Near-bed horizontal water particle velocity
U_q	Mean of the n largest velocity values
C	Wave celerity (wave speed)
n	Depth-related parameter ($n = \bar{H}/d$)

Statistical quantities

$f(x)$	Probability density function (PDF) of variable x
$g(y)$	PDF of transformed variable y
X	Primary random variable
Y	Derived random variable
J	Jacobian of transformation
N	Number of bins in PDF discretization
NRMSE	Normalized Root Mean Square Error

Subscripts and superscripts

rms	Root mean square
s	Significant value
0	Deep-water condition

References

- Auel C, Albayrak I, Sumi T, Boes RM, "Sediment transport in high-speed flows over a fixed bed: 2. Particle impacts and abrasion prediction", *Earth Surface Processes and Landforms*, 2017, 42 (9), 1384-1396.
<https://doi.org/10.1002/esp.4132>
- Dean RG, Dalrymple RA, "Water wave mechanics for engineers and scientists", World Scientific Publishing, Singapore, 1991.
- Dong P, Zhang K, "Intense near-bed sediment motions in waves and currents", *Coastal Engineering*, 2002, 45 (2), 75-87.
[https://doi.org/10.1016/S0378-3839\(02\)00040-6](https://doi.org/10.1016/S0378-3839(02)00040-6)
- Elfrink B, Hanes DM, Ruessink BG, "Parameterization and simulation of near bed orbital velocities under irregular waves in shallow water", *Coastal Engineering*, 2006, 53 (11), 915-927.
<https://doi.org/10.1016/j.coastaleng.2006.06.002>
- Grossmann F, Hurther D, van Der Zanden J, Cáceres I, Sánchez-Arcilla A, Alsina JM, "Near-bed sediment transport during offshore bar migration in large-scale experiments", *Journal of Geophysical Research: Oceans*, 2022, 127 (5), e2021JC017756.
<https://doi.org/10.1029/2021JC017756>
- Kawamata S, Kobayashi M, "Empirical formulas for

- near-bed wave orbital velocity parameters involved in maximum wave load in random wave trains", *Ocean Engineering*, 2023, 276, 114133.
<https://doi.org/10.1016/j.oceaneng.2023.114133>
- Khadami F, Suprijo T, Hidayat AR, Radjawane IM, Tarya A, Napitupulu G, "Near-Bed Flow Dynamics and Bed Shear Stress in a Mangrove-Vegetated Estuary", *Proceedings of the IOP Conference Series: Earth and Environmental Science*, February, 2025.
- Khan-Mozahedy ABM, Muñoz Pérez JJ, Neves MG, "Monitoring of the water particle velocity field near the seabed under different wave and tidal scenarios: a real case", *Proceedings of the 5th International Conference on Water and Flood Management*, Dhaka, Bangladesh, March, 2015.
- Li A, Melville BW, Yang Y, Zhou S, He F, Shamseldin AY, Zhang G, "The semi-empirical model for critical bed shear stress in the local scour hole downstream of a submerged structure based on turbulent velocity distribution", *Applied Ocean Research*, 2024, 153, 104238.
<https://doi.org/10.1016/j.apor.2024.104238>
- Liu GF, Wu JX, Wang YY, "Near-bed sediment transport in a heavily modified coastal plain estuary", *International Journal of Sediment Research*, 2014, 29 (2), 232-245.
[https://doi.org/10.1016/S1001-6279\(14\)60039-1](https://doi.org/10.1016/S1001-6279(14)60039-1)
- Massel SR, "Ocean surface waves: their physics and prediction", *World Scientific Publishing*, Singapore, 1996.
- Pu JH, "Velocity profile and turbulence structure measurement corrections for sediment transport-induced water-worked bed", *Fluids*, 2021, 6 (2), 86.
<https://doi.org/10.3390/fluids6020086>
- Song JB, Wu YH, "Statistical distribution of water-particle velocity below the surface layer for finite water depth", *Coastal Engineering*, 2000, 40 (1), 1-19.
[https://doi.org/10.1016/S0378-3839\(99\)00062-9](https://doi.org/10.1016/S0378-3839(99)00062-9)
- Sultan NJ, "Irregular wave-induced velocity in shallow water", *Technical Report No. CERC-92-9*, US Army Corps of Engineers, Washington DC, US, 1992.
- You ZJ, "The statistical distribution of near-bed wave orbital velocity in intermediate coastal water depth", *Coastal Engineering*, 2009, 56 (8), 844-852.
<https://doi.org/10.1016/j.coastaleng.2009.04.005>
- Yu Z, Loewen M, Zhou Y, Guo Z, Baki AB, Zhang W, "Continuous near-bed movements of microplastics in open channel flows: Statistical analysis", *Environmental Science & Technology*, 2025, 59 (3), 1835-1843.
<https://pubs.acs.org/doi/10.1021/acs.est.4c13351>
- Zhao C, Fang H, Ouro P, Stoesser T, Dey S, "Response of bedload and bedforms to near-bed flow structures", *Journal of Hydraulic Engineering*, 2024, 150 (1), 04023060.
<https://doi.org/10.1061/JHEND8.HYENG-13618>
- Zhou Z, Ge J, Van Maren DS, Luan H, Guo W, Ma J, Ding P, "Multi-year observations of near-bed hydrodynamics and suspended sediment at the core of the estuarine turbidity maximum of the Changjiang Estuary", *Earth System Science Data*, 2025, 17 (3), 917-935.
<https://doi.org/10.5194/essd-17-917-2025>